Map Projection Aspects

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ICA European Symposium on CARTOGRAPHY
VIENNA, 10–12 NOVEMBER 2015

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Definitions ...

The aspect of a map projection is ...

Why the aspect is important?

The choice of a suitable aspect enables an arbitrary area on the globe to be located with smaller distortions.

The successful application of various aspects of a projection is knowledge of the distortion distribution of the projection.

Herz (1885)
Hammer (1889)
Vital (1903)
Zöppritz and Bludau (1912)

Maurer, H. (1925): Schiefachsige und querachsige Karten-Abbildungen, Zeitschift für Vermessungswesen, Band LIV, Heft 7 und 8, 142–144.

Maurer, H. (1935): Ebene Kugelbilder: Ein Linnésches System der Kartenentwürfe, Petermanns Mitteilungen, Ergänzungsheft no. 221.

Maurer, H. (1935)

- *Earth-axis* is sometimes referred to as normal (*normal*), pole-standing (*polständig*), polar-axis (*polachsig*), straight-axis (*rechtachsig*), polar projection (*Polar-Projektion*), equatorial projection (*Äquator-Projektion*)
- *Transverse-axis* is sometimes referred to as transverse (*transversal*), equator-standing (*äquatorständig*), equator-axis (*äquatorachsig*), meridian projection (*Meridian-Projektion*), equatorial projection (*Äquator-Projektion*)
- *Oblique-axis* is sometimes referred to as horizontal (*horizontal*), zenithal (*zenital*), inter-standing (*zwischenständig*), inclined-axis (*schrägachsig*), meridian projection (*Meridian-Projektion*), transversal (*transversal*), transverse-axis (*querachsig*).

Lee, L. P. (1944): The Nomenclature and Classification of Map Projections, Empire Survey Review, No. 51, Vol. VII, 190–200.

Cylindric: projections in which the meridians are represented as a system of equidistant parallel straight lines, and the parallels by a system of parallel straight lines at right angles to the meridians.

Conic: projections in which the meridians are represented as ...

Azimuthal: projections in which the meridians are represented as ...

No cylinders, no cones, ... ?!

"No reference has been made in the above definitions to cylinders, cones or planes. The projections are termed cylindric or conic because they can be regarded as developed on a cylinder or cone, as the case may be, but it is as well to dispense with picturing cylinders and cones, since they have given rise to much misunderstanding.

Particularly is this so with regards to the conic projections with two standard parallels: they may be regarded as developed on cones, but they are cones which bear no simple relationship to the sphere.

In reality, cylinders and cones provide us with convenient descriptive terms, but little else."

By Lee: In the case of the general conical projections,

- the *direct aspect* is that in which the axis of the cylinder, cone or plane coincides with the polar axis of the sphere;
- the *transverse aspect* that in which the axis of the cylinder, cone or plane lies in the plane of the equator; and
- the *oblique aspect* that in which the axis has any other position.

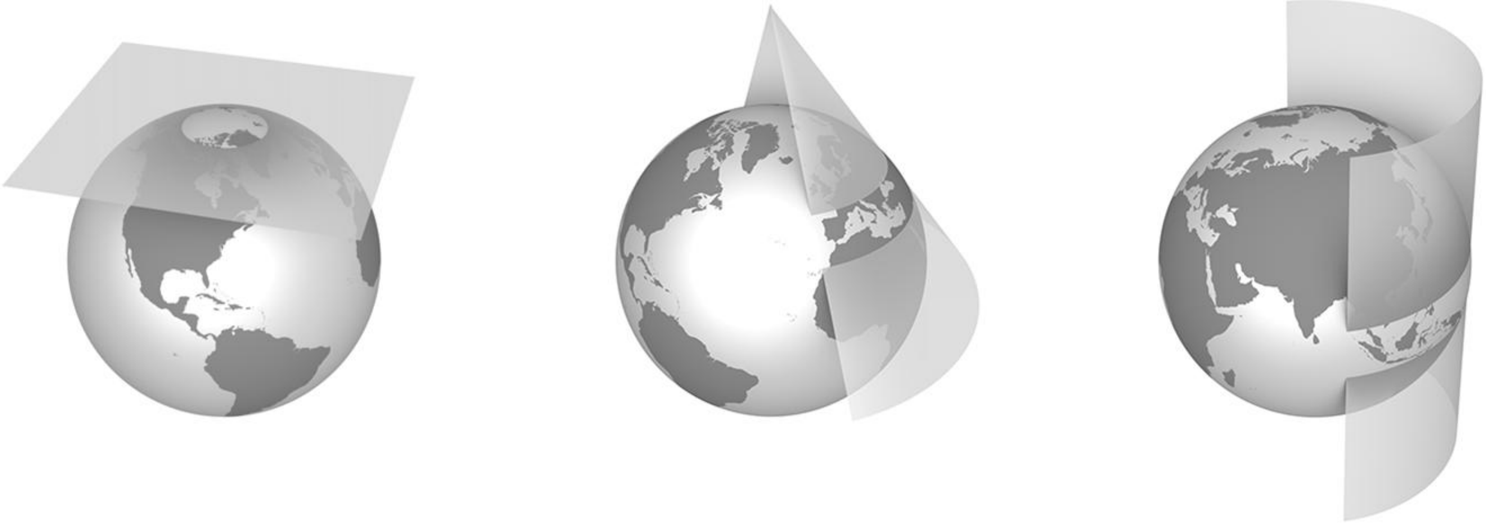
It is difficult to extend the definitions to cover the non-conical projections, ...

A rather curious position arises in the case of some of those projections which fall into more than one of the graticule groups given above.

Stereographic - considered as an azimuthal projection, the direct aspect is that in which the point of tangency is the pole, and the transverse aspect that in which the point of tangency is at the equator

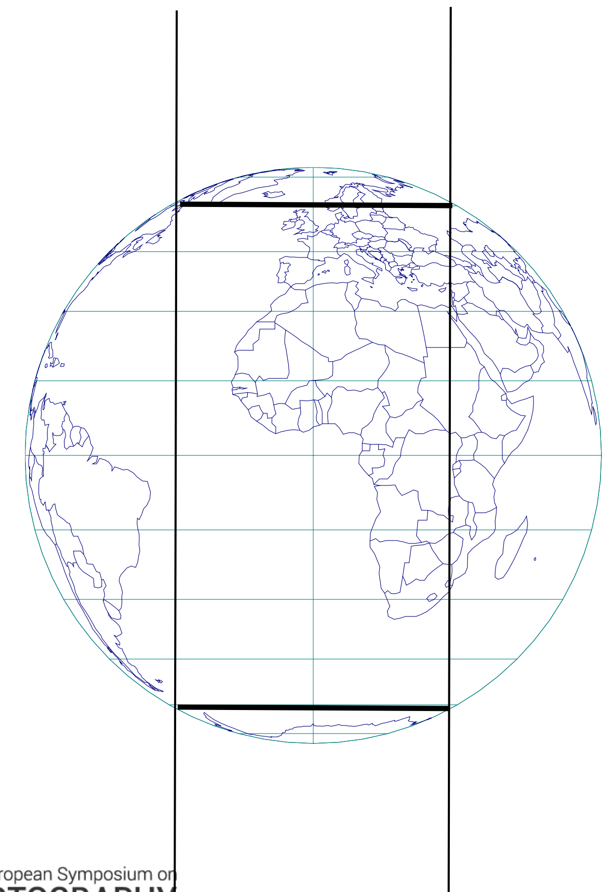
Stereographic - considered as a polyconic, the direct aspect is that in which the point of tangency is at the equator, and the transverse aspect that in which the point of tangency is the pole.

Similar remarks apply to the orthographic.



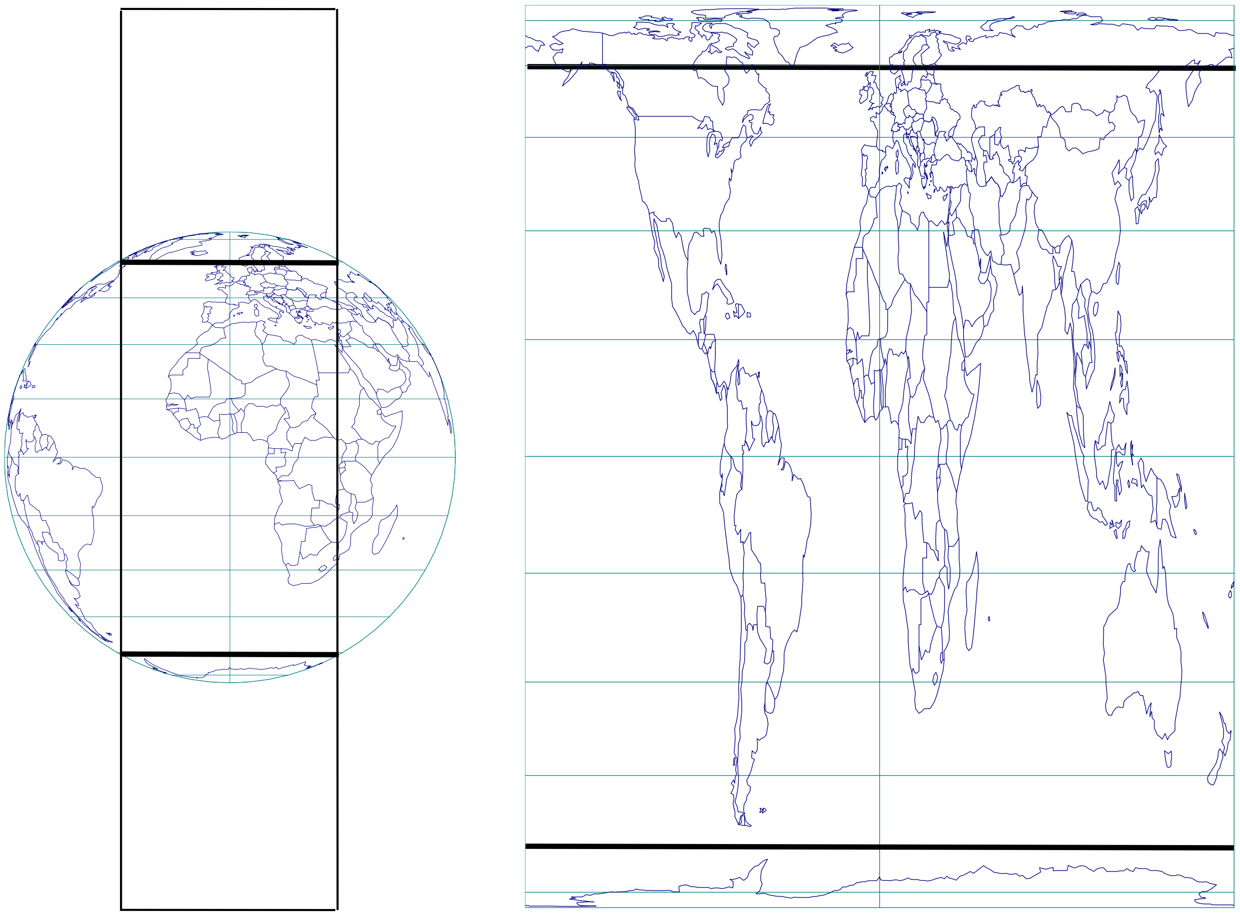
Is the normal aspect of Mercator projection with two standard parallels a secant projection?

No, it isn't.



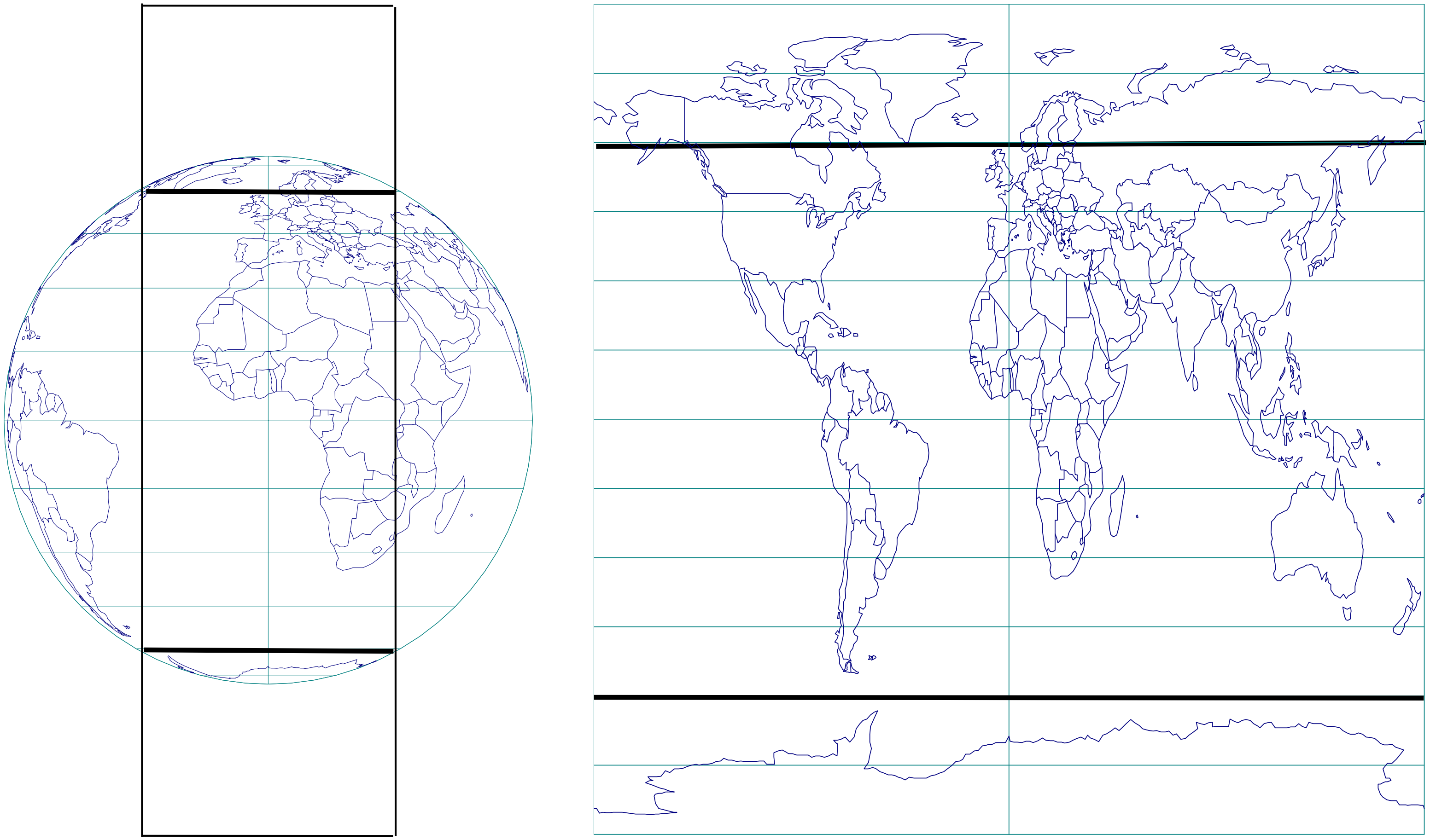
Is the normal aspect of equal-area cylindrical projection with two standard parallels a secant projection?

No, it isn't.



Is the normal aspect of equidistant cylindrical projection with two standard parallels a secant projection?

No, it isn't.



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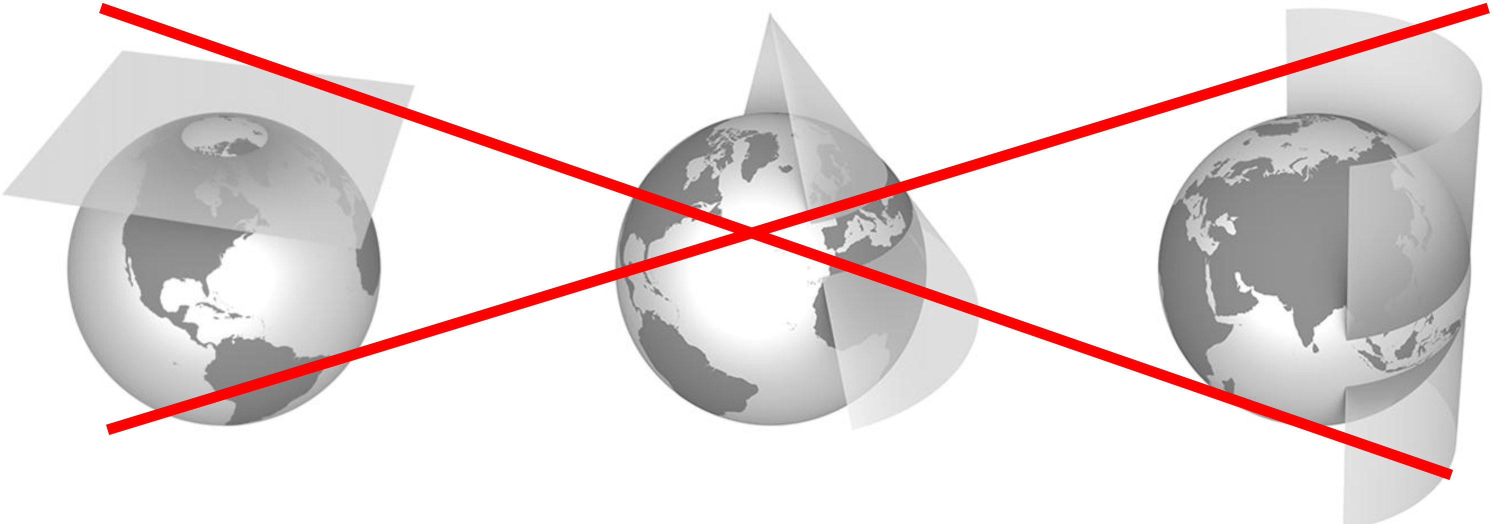
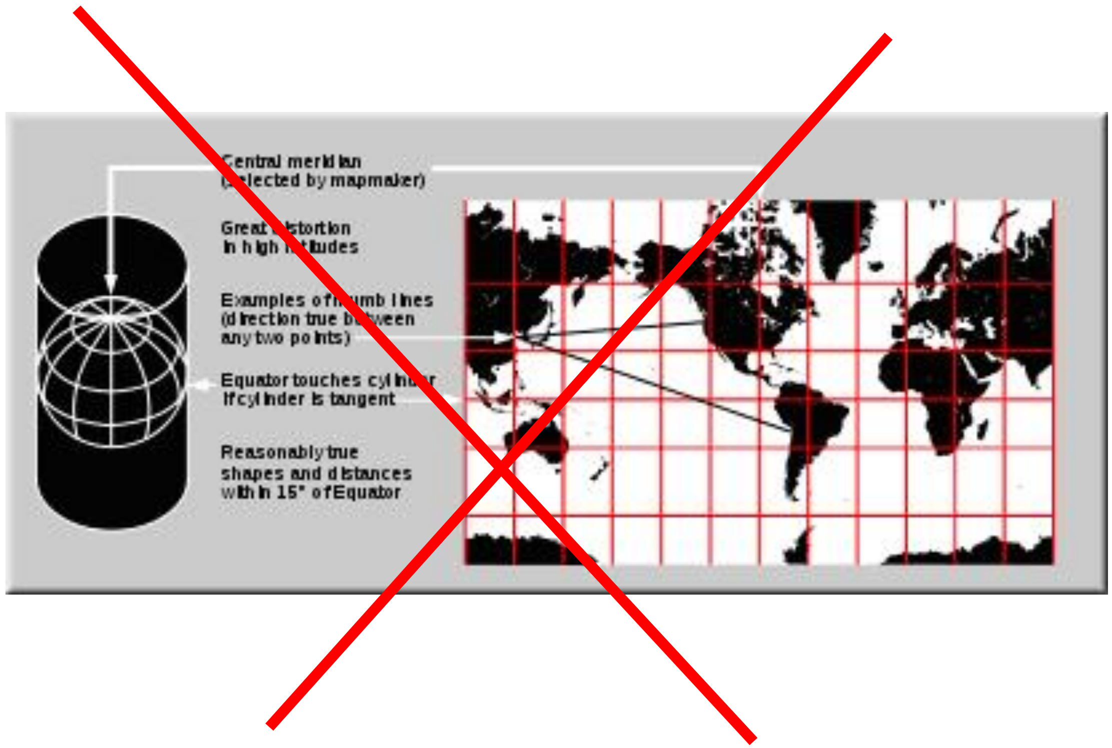
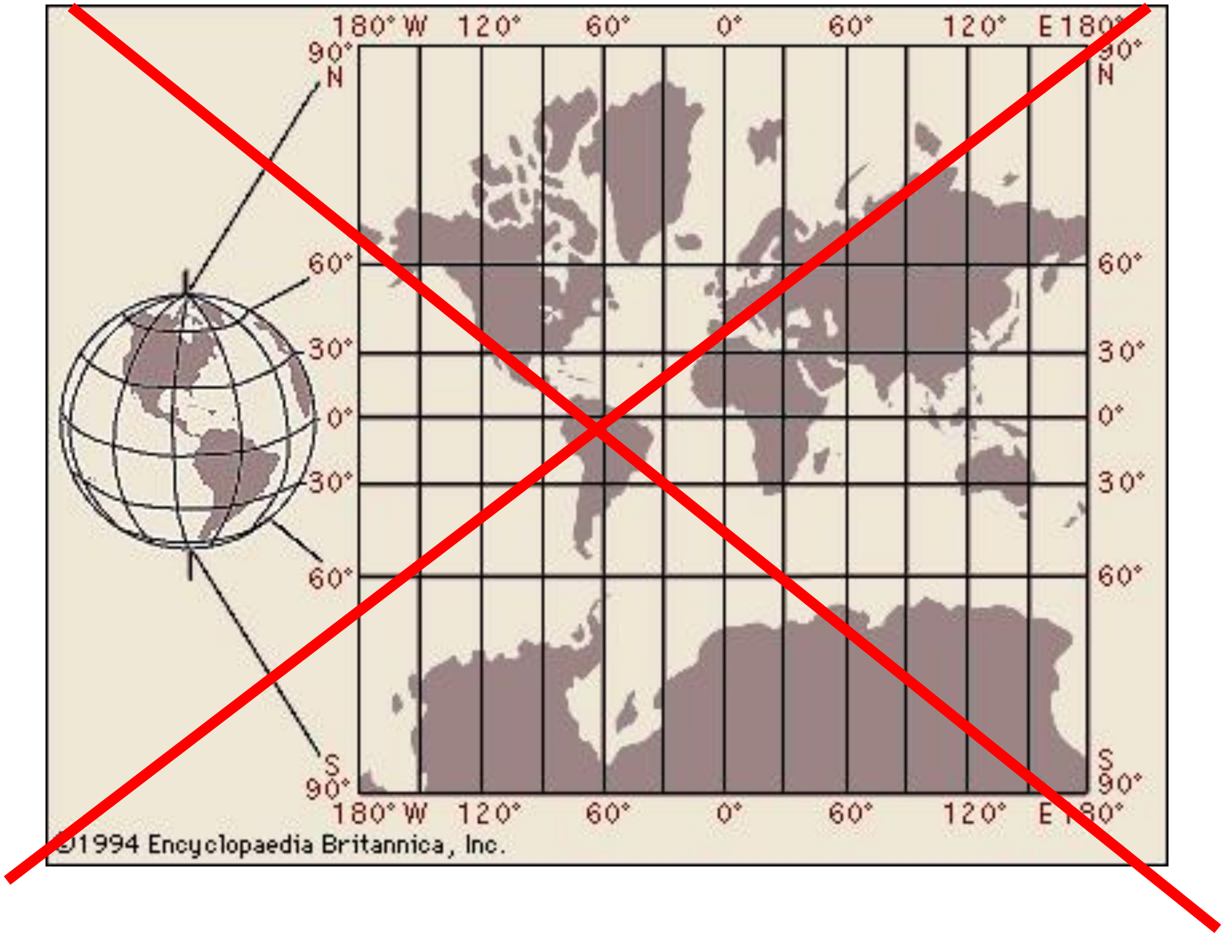


Please remember

Ther is no cylinder in derivation of cylindrical projections, with the exception to perspective cylindrical projections.



Please remember



Please remember

Kavrayskiy (1958)
Hoschek (1969)
Wray (1974)
Maling (1980)
Beineke (1983)
Kuntz (1983)
Čapek (1986)
Snyder (1987)
Snyder and Voxland (1989)
Bugayevskiy and Snyder (1995)
De Genst and Canters (1996)
Encyclopaedic Dictionary of Cartography in 25 Languages (Neumann 1997)
Canters (2002)
Grafarend and Krumm 2006
Fenna (2007)
Lapaine and Usery (2014)
...

Question:

Is it possible to define projection aspect without referring to auxiliary surfaces?

Geographic parameterization

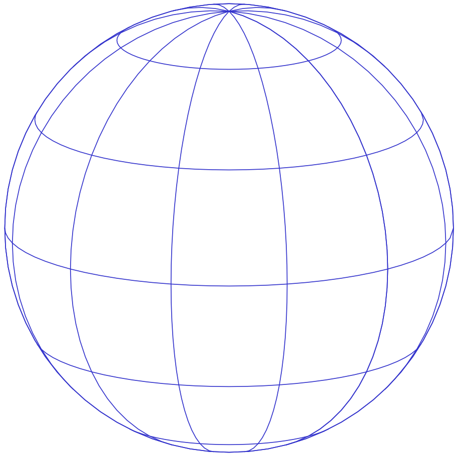
$$(\varphi, \lambda) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \times [-\pi, \pi] \to (X, Y, Z) \in \mathrm{R}^3$$

$$X = R\cos\varphi\cos\lambda \qquad Y = R\cos\varphi\sin\lambda \qquad Z = R\sin\varphi$$

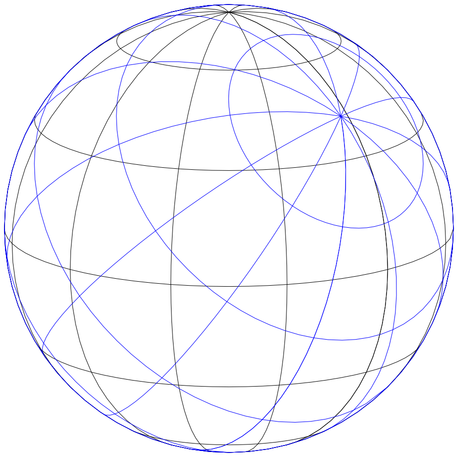
Pseudogeographic parameterization

$$(\varphi', \lambda') \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \times [-\pi, \pi] \to (X', Y', Z') \in \mathrm{R}^3$$

$$X' = R\cos\varphi'\cos\lambda' \quad Y' = R\cos\varphi'\sin\lambda' \quad Z' = R\sin\varphi'$$



(Visualization of) geographic parameterization of a sphere



Geographic (black) and pseudogeographic (blue) parameterization of a sphere

$$\sin\varphi' = \sin\varphi_P \sin\varphi + \cos\varphi_P \cos\varphi \cos(\lambda - \lambda_P)$$

$$\tan\lambda' = \frac{\sin(\lambda - \lambda_P)}{\cos\varphi_P \tan\varphi - \sin\varphi_P \cos(\lambda - \lambda_P)}$$

Two parameters only?

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

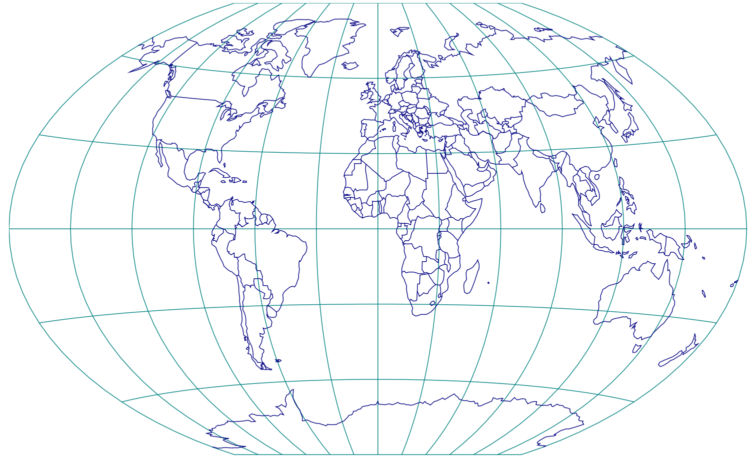
Orthogonal matrix

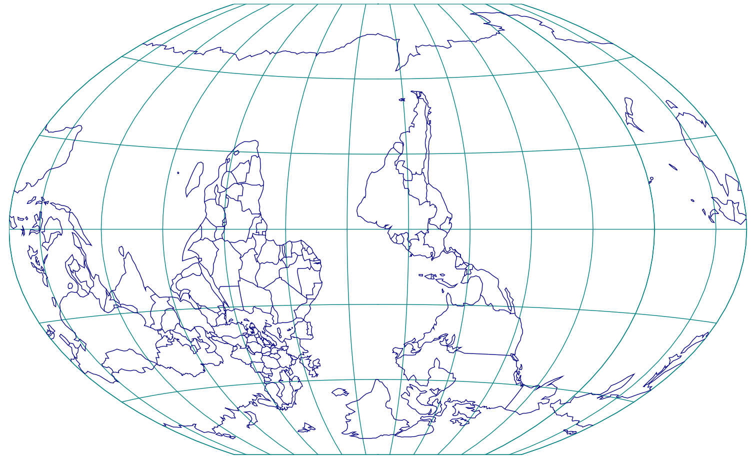
$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\theta_P & \sin\theta_P & 0 \\ -\sin\theta_P & \cos\theta_P & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin\varphi_P & 0 & -\cos\varphi_P \\ 0 & 1 & 0 \\ \cos\varphi_P & 0 & \sin\varphi_P \end{bmatrix} \begin{bmatrix} \cos\lambda_P & \sin\lambda_P & 0 \\ -\sin\lambda_P & \cos\lambda_P & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\cos\varphi'\cos\lambda' = \sin\varphi_P\cos\varphi\cos(\lambda-\lambda_P)\cos\theta_P - \cos\varphi_P\sin\varphi\cos\theta_P + \cos\varphi\sin(\lambda-\lambda_P)\sin\theta_P$$

$$\cos\varphi'\sin\lambda' = -\sin\varphi_P\cos\varphi\cos(\lambda-\lambda_P)\sin\theta_P + \cos\varphi_P\sin\varphi\sin\theta_P + \cos\varphi\sin(\lambda-\lambda_P)\cos\theta_P$$

$$\sin\varphi' = \cos\varphi_P\cos\varphi\cos(\lambda-\lambda_P) + \sin\varphi_P\sin\varphi$$





Two world maps in the normal aspect of a Winkel Tripel projection

Three world maps in the transverse aspect of a Winkel Tripel projection

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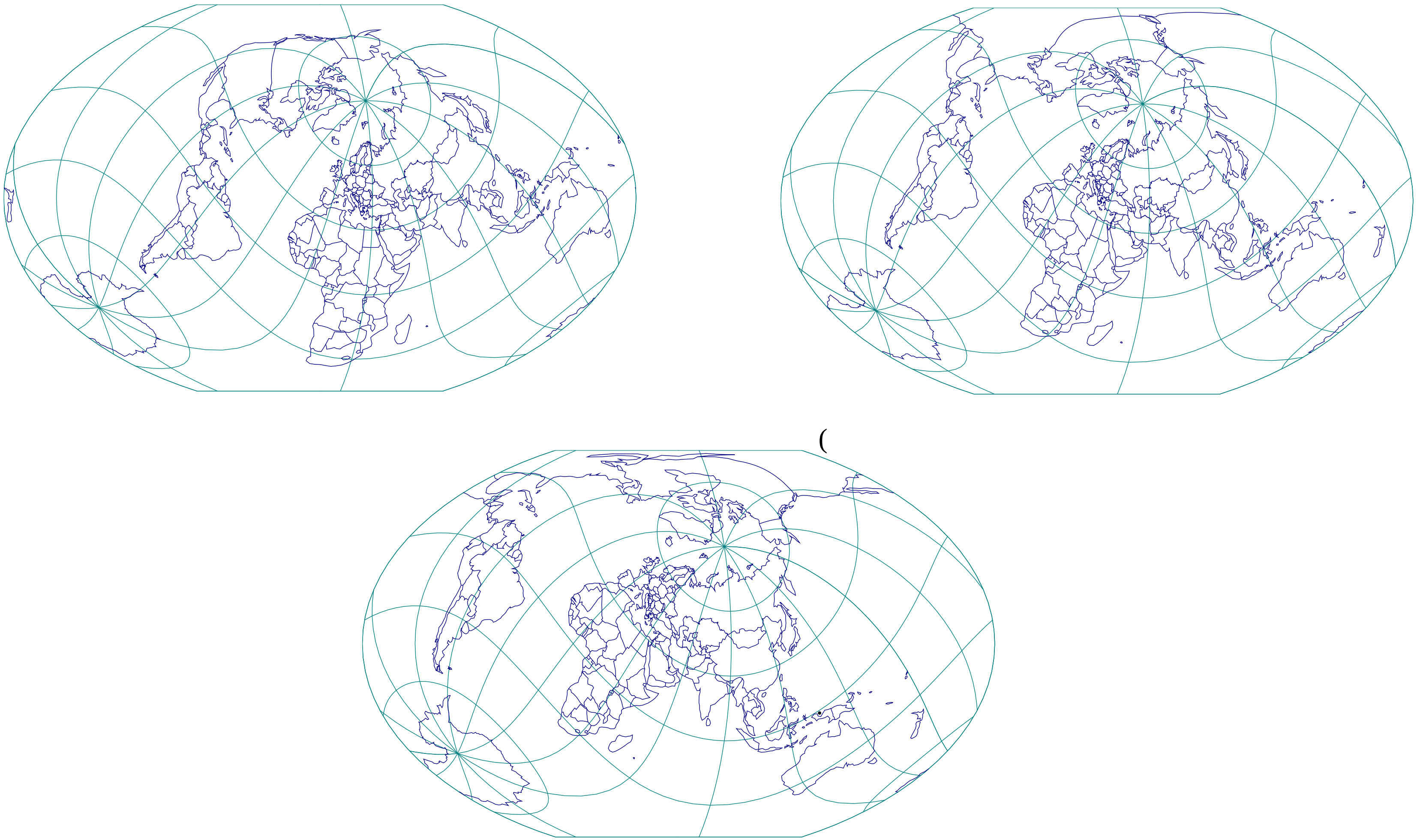

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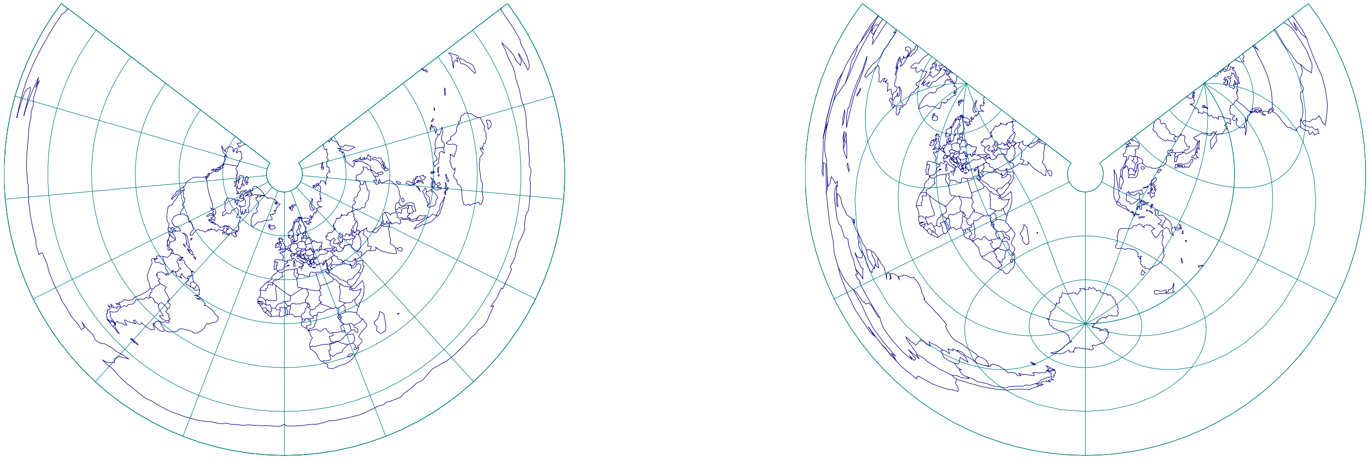
World map in the oblique aspect of a Winkel Tripel Projection



Normal and transverse conic projection
(equidistant along meridians).
Illustration of problematic choices of polar or equatorial aspects.

The *projection aspect* is the position of the projection axis in relation to the geographic sphere parameterization axis.

The *projection axis* is the axis of pseudogeographic parameterization of a sphere, based on which the basic equations of map projection are defined.

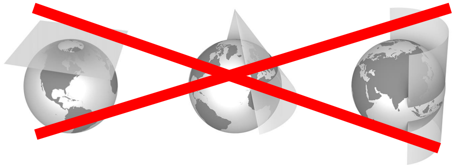
The aspect can be normal, transverse or oblique.

The *normal aspect* is the aspect in which the projection axis coincides with the geographic sphere parameterization axis.

The *transverse aspect* is the aspect in which the projection axis is perpendicular to the geographic sphere parameterization axis.

The *oblique aspect* is neither normal nor transverse.

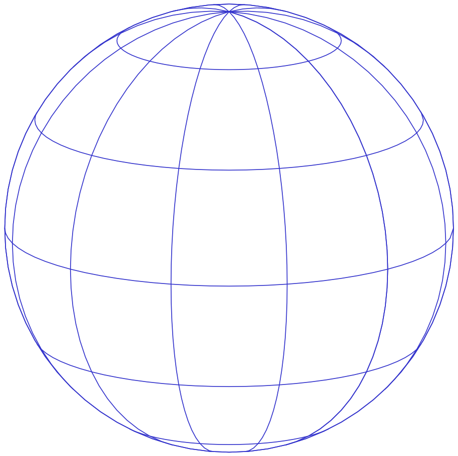
Conclusion

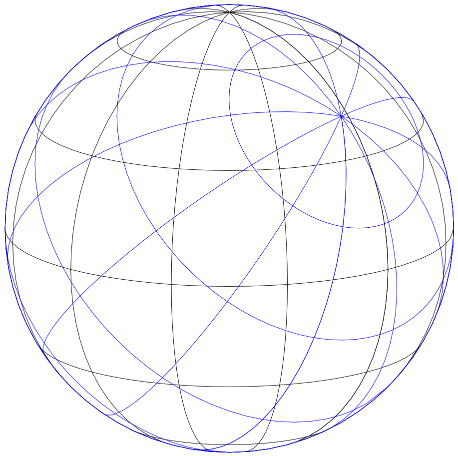


- There is no need to use secant types of projections, because they generally can give a wrong perception.
- The aspect is important because its choice enables an area on the globe to be located with smaller distortions.
- There is no generally accepted definition of a projection aspect, even though the term itself is widely used.

Conclusion

- Transition from one aspect to another is equivalent to the rotation of a spatial coordinate system unambiguously defined by 3 parameters, no 2.
- We do not recommend the definition (e.g. polar, equatorial) accoridng to which the aspect is the representation of an area in the central part of the map, because of its ambiguity.
- We propose a rigorous, mathematically based definition of map projection aspects.





Thank you for your attention