SELÇUK ÜNİVERSİTESİ

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The Web Mercator Projection: A Cartographic Analysis

İbrahim Öztuğ Bildirici

Selcuk University
Konya,Turkey

bildirici@selcuk.edu.tr

Introduction

- Web-based mapping services are using a modified version of Mercator Projection that is called Web Mercator.
- The WGS84 ellipsoid is transformed to the map by using spherical projection equations.
- The resulting projection is no more conformal.
- Neither Mercator nor the Web Mercator is not suitable for portraying the whole earth.

Map Projection

- A transformation from the reference surface to the map (Forward Transformation)

$$x = x(\varphi, \lambda)$$
$$y = y(\varphi, \lambda)$$

  - Reference surface: Ellipsoid or sphere
- Inverse transformation (from the map to the reference surface)

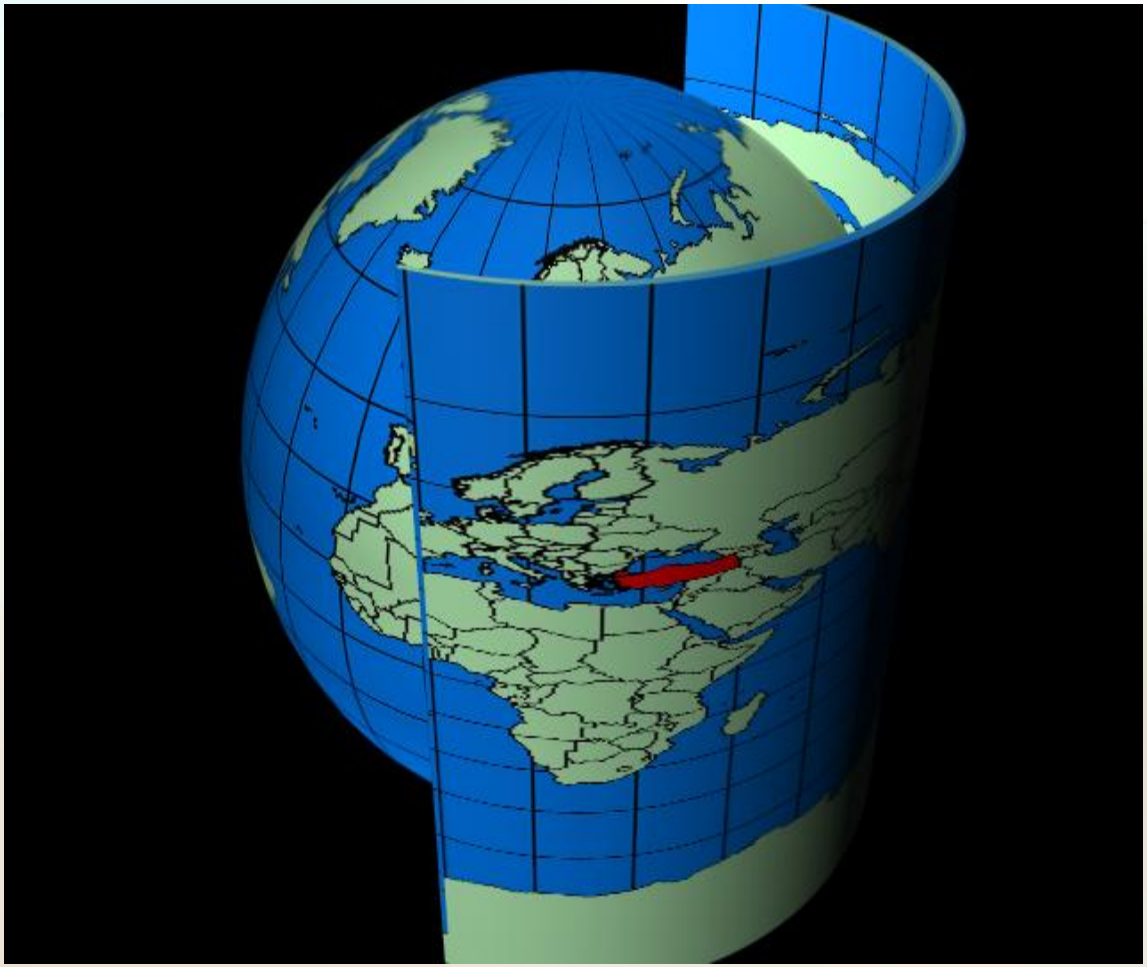
$$\varphi = \varphi(x, y)$$
$$\lambda = \lambda(x, y)$$

Cylindrical Projections

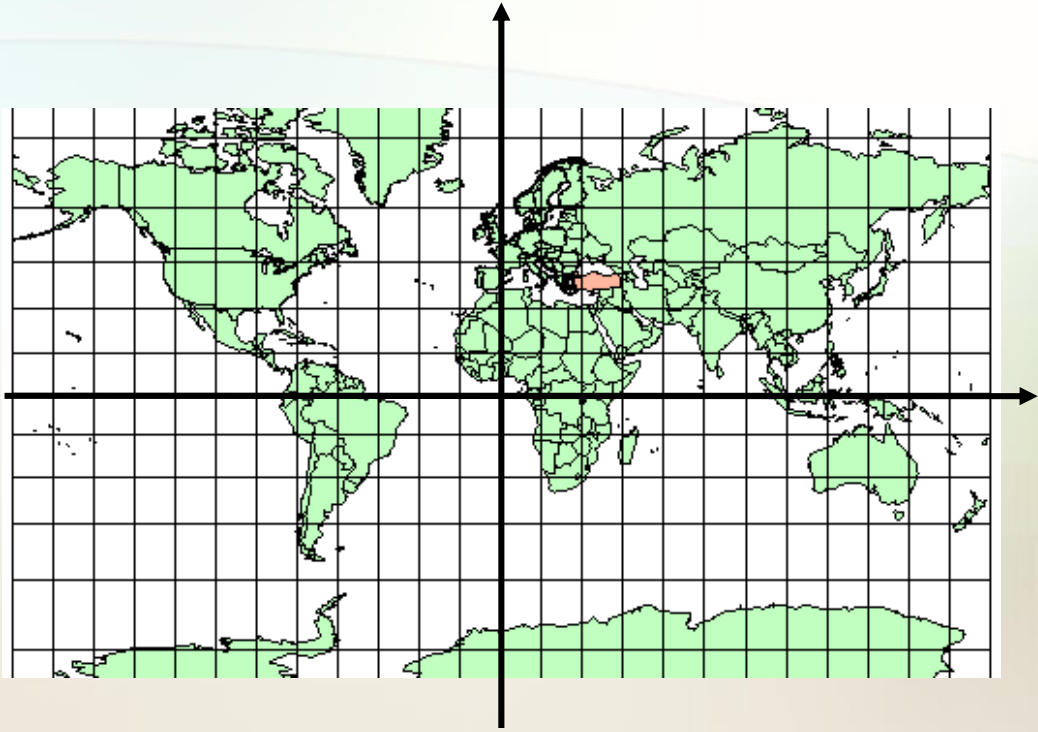
Sphere

→ Cylinder

→ Plane



- The rectangular earth in cylindrical projection enables a suitable coordinate system.
- High distortion occurs towards poles!
- A conformal projection is required to portray the objects of the real world similarly.
- This is important when presenting man-made objects such as buildings in aerial imagery.



$$x = f_1(\lambda)$$

$$y = f_2(\varphi)$$

Mercator

- One of the well-known projections
- Cylindrical and conformal.
- Rhumb lines are depicted as straight lines.
- One or two standard parallels
- areas towards poles are distorted too much.
  - At 60° latitude areas are 4 times exaggerated.
- Polar areas can not be shown.
- Widely used.

- Forward transformation:

$$x = R\lambda$$

$$y = R \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$$

Spherical

One standard parallel

- Linear distortion:

$$m = \frac{1}{\cos\varphi}$$

- Forward transformation:

$$x = a\lambda$$

$$y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{\frac{e}{2}}$$

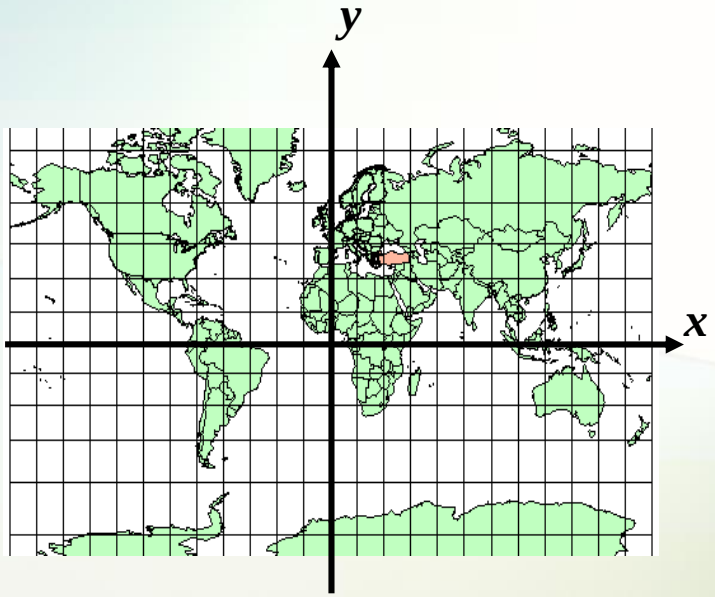
Ellipsoidal

One standard parallel

- Linear distortion:

$$m = \frac{a}{N\cos\varphi}$$

Web Mercator



- WGS84 ellipsoid is transformed to the map plane with spherical equations.
- No more conformal

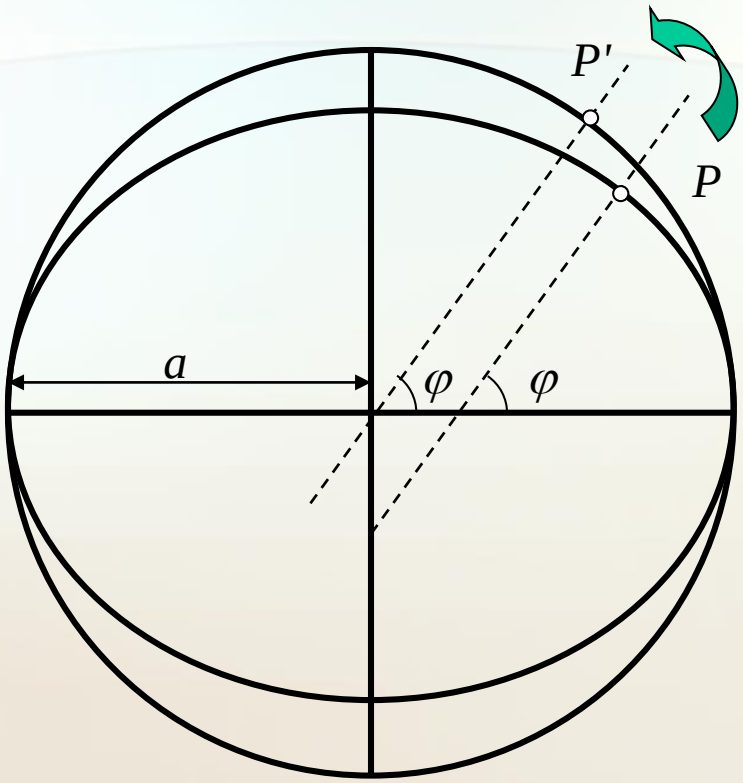
$$x = a\lambda$$

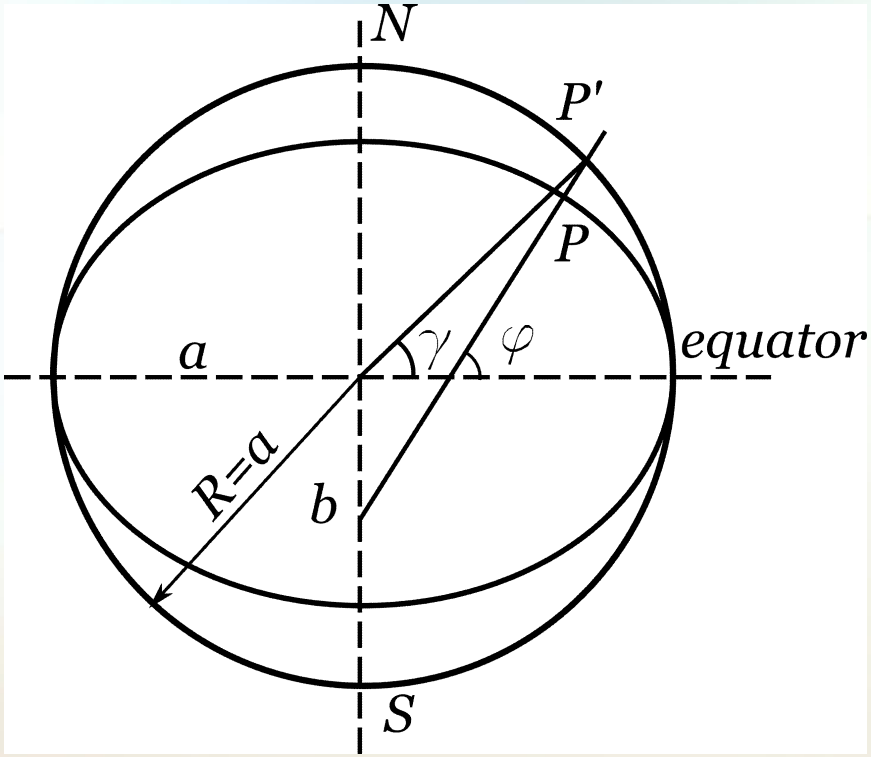
$$y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1-}{1}\right)^{\frac{}{2}}$$

$$x = a\lambda$$

$$y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$$

Latitude?

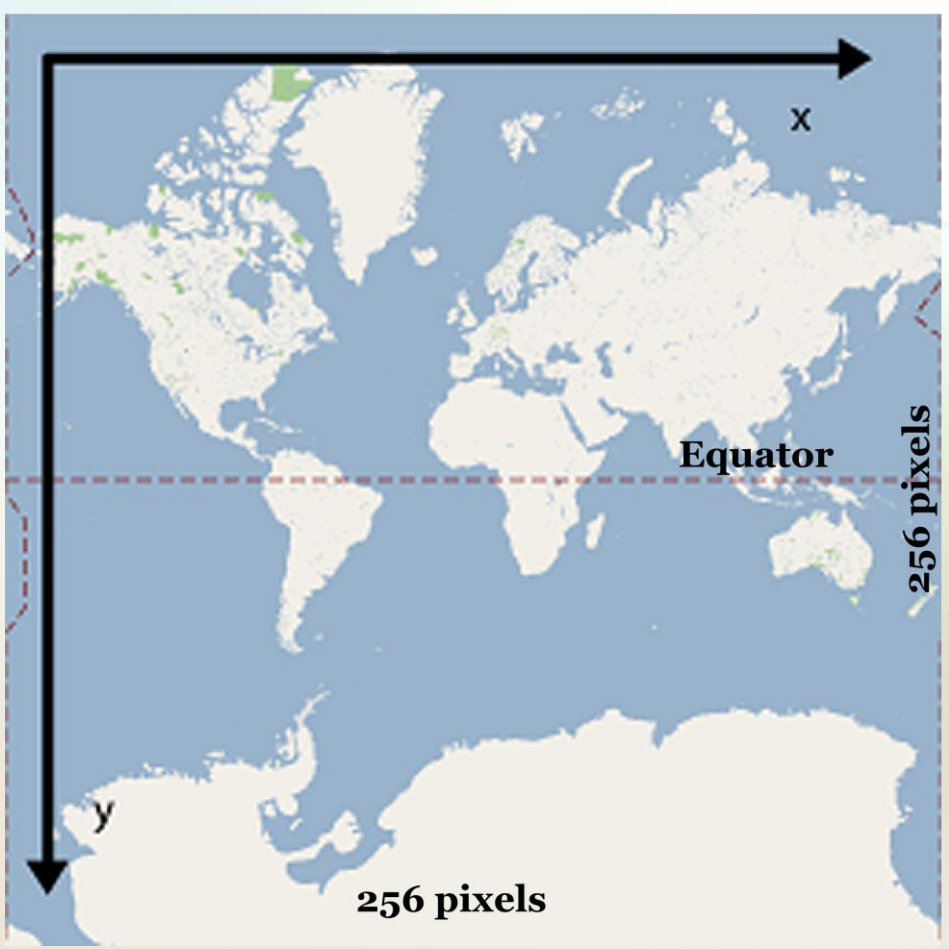



N
P'
P
a
γ
φ
equator
R=a
b
S

$$x = \frac{128}{\pi} 2^n (\lambda + \pi)$$

$$y = \frac{128}{\pi} 2^n \left(\pi - \ln \tan \left(\frac{\pi}{4} + \frac{\varphi}{2} \right) \right)$$

Coordinate system is shifted and scaled!

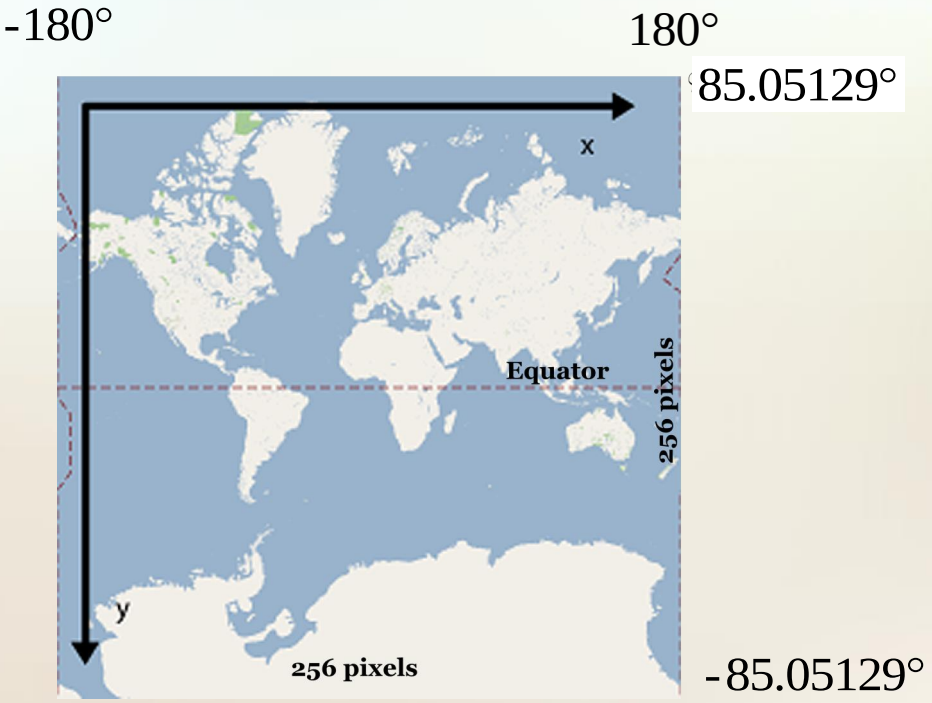


n=0

$$\text{setting } y = 0, n = 0$$

$$\ln \tan\left(\frac{\pi}{4} + \frac{\varphi_{\max}}{2}\right) = \pi$$

$$\varphi_{\max} = 2 \arctan e^{\pi} - \frac{\pi}{2} = \pm 85.05129°$$



- The web mapping services use zoom levels up to 18.
- The spatial resolution of 1 pixel ranges from 156 km at zoom level 0 to 0.60 m at zoom level 18.

$$1 pixel = \frac{\pi a}{2^{n+7}} meters$$

Inverse Transformation

$$\varphi = 2\arctan e^{\pi - \frac{\pi y}{2^{n+7}}} - \frac{\pi}{2}$$

$$\lambda = \frac{\pi x}{2^{n+7}} - \pi$$

Projection Distortions

- For this purpose metric projection equations should be used.

$$x = a\lambda$$

$$y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$$

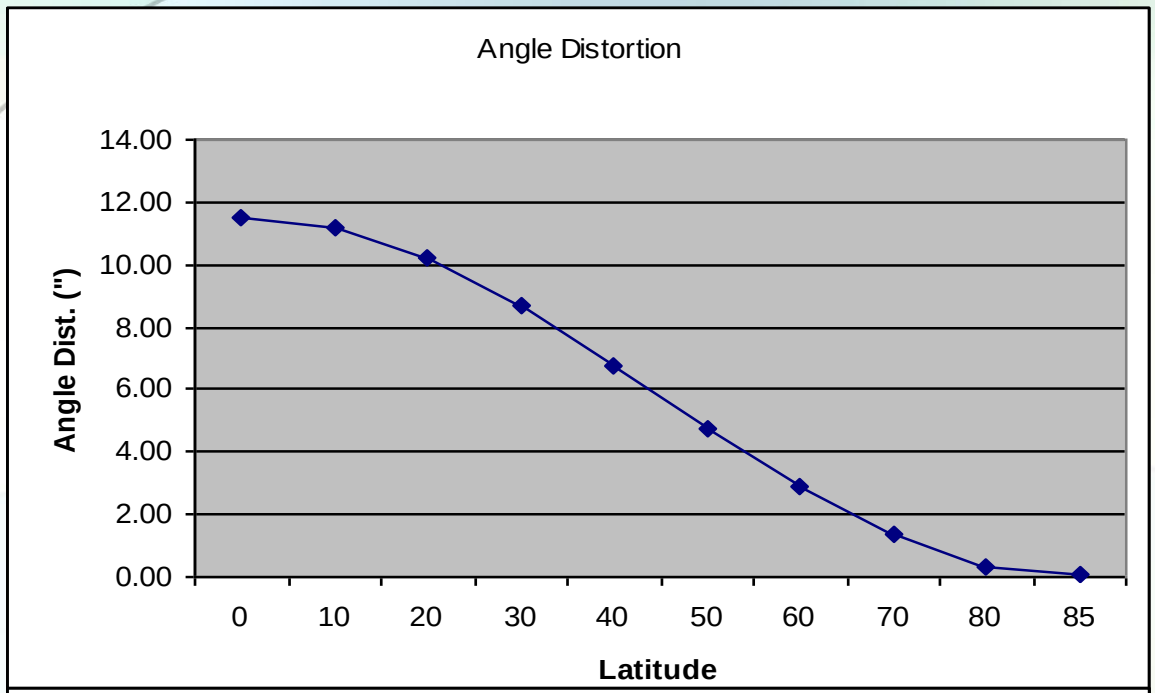
- Distortions along meridians and parallels:

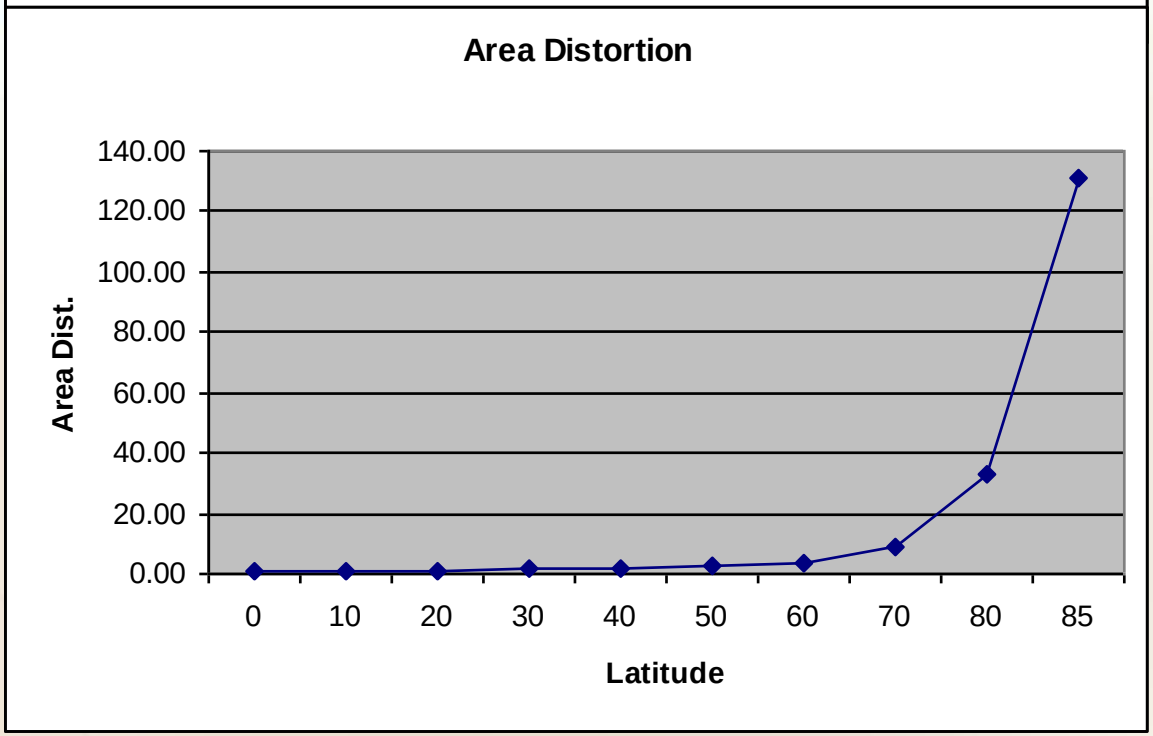
$$h = \frac{1}{M}\frac{dy}{d\varphi} = \frac{a}{M\cos\varphi}$$

$$k = \frac{1}{N\cos\varphi}\frac{dx}{d\lambda} = \frac{a}{N\cos\varphi}$$

Distortions

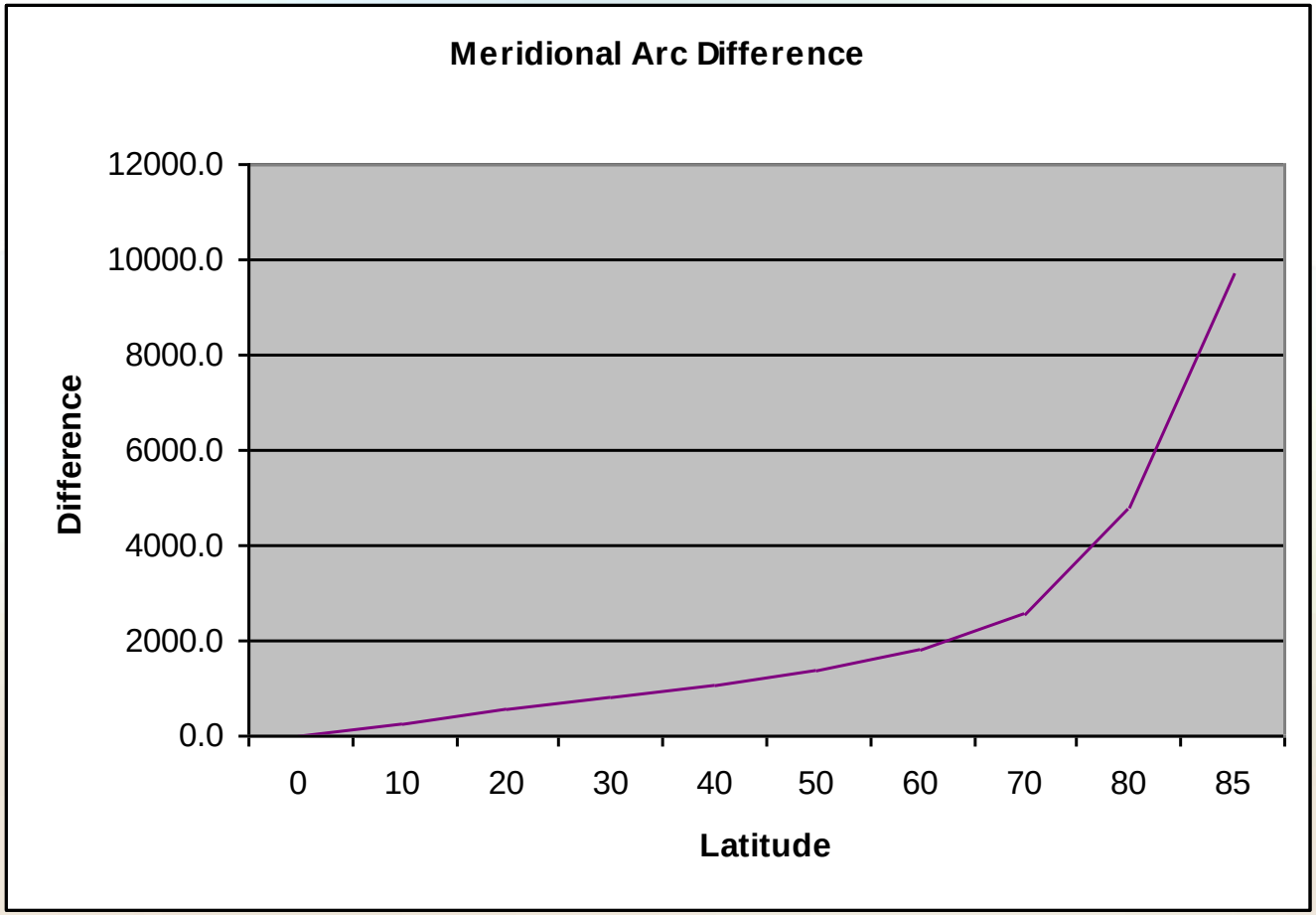
φ	h	k	p	ω
0°	1.006739	1.000000	1.006739	11' 32.7"
10°	1.021961	1.015324	1.037621	11' 11.9"
20°	1.070092	1.063761	1.138322	10' 11.9"
30°	1.159566	1.153734	1.337830	8' 40.0"
40°	1.308756	1.303601	1.706096	6' 47.1"
50°	1.556989	1.552665	2.417482	4' 46.8"
60°	1.998334	1.994973	3.986623	2' 53.6"
70°	2.917448	2.915150	8.504798	1' 21.3"
80°	5.741212	5.740046	32.95482	0' 21.0"
85°	11.43612	11.43554	130.7782	0' 5.3"


Angle Distortion
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
Angle Dist. (")
0
10
20
30
40
50
60
70
80
85
Latitude


Area Distortion
140.00
120.00
100.00
80.00
60.00
40.00
20.00
0.00
Area Dist.
0
10
20
30
40
50
60
70
80
85
Latitude

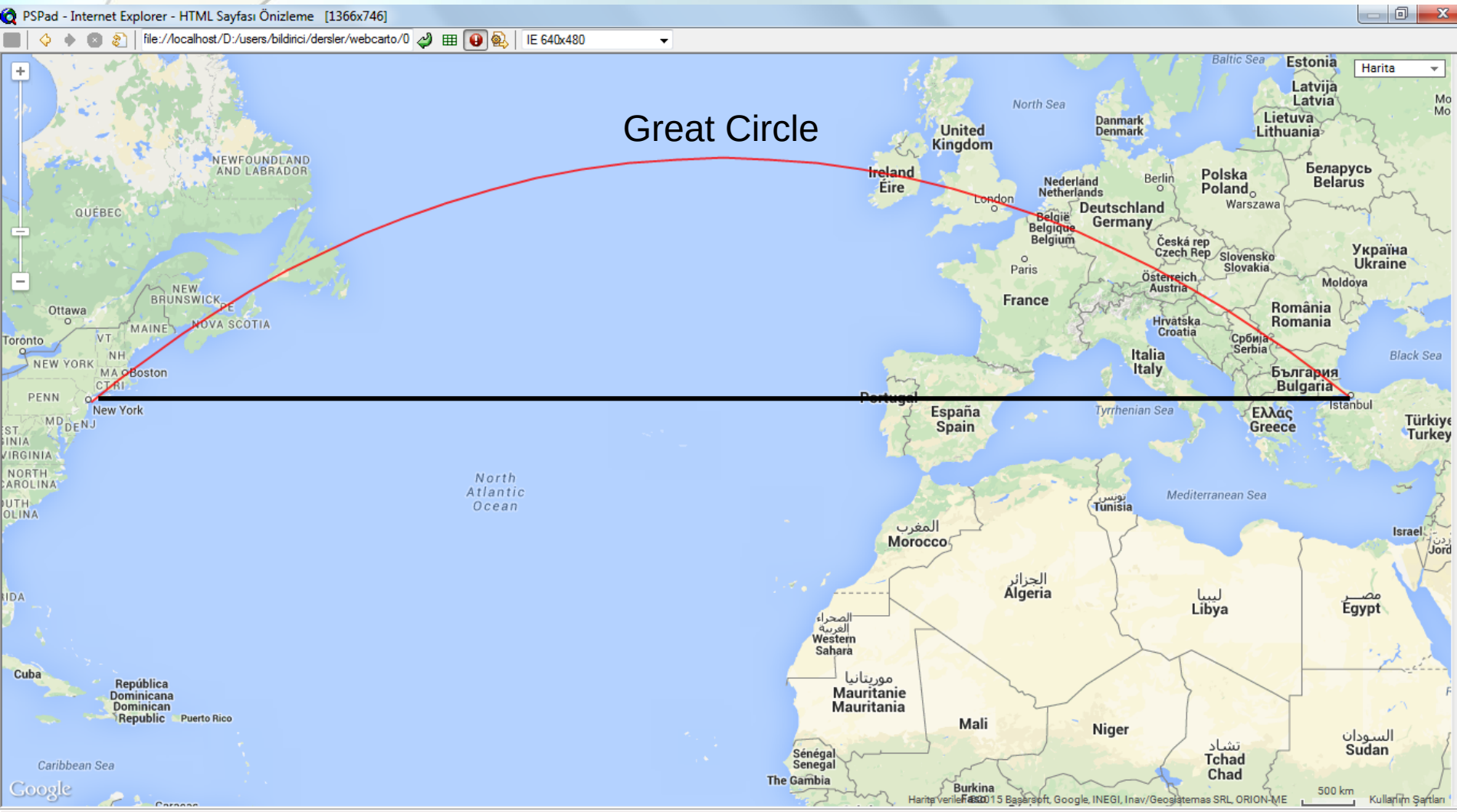
1° Meridian Arc Comparison

Latitude	WGS84 (m)	Web M. (m)	Mercator (m)	Difference (WM-M) (m)
0°	110574.389	111325.1	111312.0	13.1
10°	110611.187	113216.8	112942.0	274.8
20°	110710.615	118847.7	118310.8	536.8
30°	110860.926	129199.3	128396.9	802.4
40°	111044.261	146399.4	145318.3	1081.1
50°	111238.681	175017.9	173620.6	1397.2
60°	111420.728	226085.3	224268.1	1817.2
70°	111568.259	333556.5	330999.0	2557.5
80°	111663.201	675090.0	670299.7	4790.3
85°	111687.001	1424698.4	1414946.5	9751.9


Meridional Arc Difference
12000.0
10000.0
8000.0
6000.0
4000.0
2000.0
0.0
Difference
0
10
20
30
40
50
60
70
80
85
Latitude

An Alternative: Virtual Globe

- Polar areas can be mapped
- Less, or no distortions (depending on the view point)
- Great circles are represented correctly.


PSPad - Internet Explorer - HTML Sayfası Önizleme [1366x746]
file://localhost/D:/users/bildirici/dersler/webcarto/0
IE 640x480
Great Circle
Harita
New York
Boston
Ottawa
Toronto
Portugal
España
Spain
France
United Kingdom
Ireland
Éire
London
Paris
Deutschland
Germany
Berlin
Polska
Poland
Warszawa
Беларусь
Belarus
Україна
Ukraine
Moldova
România
Romania
Italia
Italy
Ελλάς
Greece
Istanbul
Türkiye
Turkey
Tunisia
Morocco
Algeria
Libya
Egypt
Israel
Western Sahara
Mauritanie
Mauritania
Mali
Niger
Tchad
Chad
Sudan
Cuba
República Dominicana
Dominican Republic
Puerto Rico
Caribbean Sea
North Atlantic Ocean
Mediterranean Sea
Black Sea
Tyrrhenian Sea
Baltic Sea
North Sea
Estonia
Latvija
Latvia
Lietuva
Lithuania
Danmark
Denmark
Nederland
Netherlands
België
Belgique
Belgium
Česká rep
Czech Rep
Slovensko
Slovakia
Österreich
Austria
Hrvatska
Croatia
Србија
Serbia
България
Bulgaria
Ελλάς
Greece
Sénégal
Senegal
The Gambia
Burkina Faso
500 km
Google
Done


Great Circle
© 2015 Google
Image Landsat
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
US Dept of State Geographer
Görüntü Tarihi: 4/10/2013 Enlem 39.898498° Boylam -21.230041° yükseklik -4358 m göz hizası 6852.44 km
Google earth

Conclusions

- Despite being unsuitable in terms of map projection distortions, Web Mercator Projection is beneficial in terms of computer graphics and tiling system.
- The graticule appears rectangular that matches the Cartesian coordinates on the map plane.
- Virtual globe representations are another way of web-based visualization.
- Google supported earth view in the previous version of Google Maps API, and later launched Google Earth API. Both are deprecated now.
- Since globe representation is a 3D visualization not affected by map projection distortions, their use is beneficial together with 2D map.
- Such possibilities can help map users to understand changes in geometry caused by map projection.

Thank you for your attention!

bildirici@selcuk.edu.tr iobildirici@yahoo.com