The Web Mercator Projection: A Cartographic Analysis

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Abstract. The well-known Mercator projection is widely used by web-based mapping services such as Google Maps, Bing Maps, in a modified version, which is called Web Mercator. This is the simplified version of the ellipsoidal projection, which is no more conformal. In this paper the properties of this projection, especially its deviation from conformality, is focused on. Additionally web-based 3D globe visualization is suggested as a possible alternative for online mapping.

Keywords. Map projections, Web Mercator, online mapping, GI Science, virtual globes

1. Introduction

Web-based mapping services are using a new type of cylindrical map projection adapted from well-known Mercator projection that is called Web Mercator. In this projection the geographic data based on WGS84 datum is transformed to the map by using spherical projection equations. The resulting projection is no more conformal. This adaptation is only beneficial in terms of computer graphics and tiling system that makes possible to publish maps at different zoom levels. Both the ellipsoidal Mercator and the Web Mercator projections are not suitable for the representation of the whole world or large parts of the world due to high scale changes towards poles.

Buttersby et al (2014) comprehensively analyzed the Web Mercator projection. They also give numerical and visual comparisons between ellipsoidal Mercator and Web Mercator. A similar analysis is given by NGA (2014).

Adaptive composite projections can be an alternative to Web Mercator projection (Savric & Jenny 2014, Jenny 2012).

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In this paper, the non-conformality of Web Mercator is mathematically shown and a numerical distortion analysis is given. The Web Mercator and the ellipsoidal Mercator projections are compared. As an alternative solution 3D virtual globe representation is suggested. Google provided such an opportunity that is deprecated lately. This was a good opportunity together with a web-based map that helps inexperienced users to understand shape changes caused by map projections.

In the following sections the basic problem of map projections is introduced. Mercator projection and Web Mercator adaptation is mathematically explained and compared. The difference between spherical and ellipsoidal latitudes is explained. Then virtual globe representation possibility is discussed. At the end some conclusions are given.

2. The Map Projection Problem

The earth's surface is represented by maps. Since the surface is curvilinear it must be transformed to the plane. The term map projection refers to this transformation process, and defined by two functions that relate geographical coordinates on the reference surface to the coordinates on the map plane.

2.1. Reference Surface

The surface of the earth is represented either by a sphere or by an ellipsoid for mapping purposes. The geometry of the sphere is rather simple. It has only one parameter, the radius. Spherical earth radius is determined by using ellipsoid dimensions. Mostly an authalic sphere is used of which surface area equals to the surface area of the ellipsoid. For each ellipsoid an authalic sphere can be defined. For WGS84 ellipsoid the authalic sphere radius is approximately 6371km.

The earth can be represented by an ellipsoid of rotation, also called spheroid, depending on two parameters, the equatorial radius (a), and the polar radius (b). The flattening, first and second eccentricity are given by the following equations.

$$
\begin{aligned}
f &= \frac{a-b}{a} \\
e &= \sqrt{\frac{a^2-b^2}{a^2}} \\
e' &= \sqrt{\frac{a^2-b^2}{b^2}}
\end{aligned}
\qquad (1)
$$

Since the curvature is not constant on the surface, the radius along meridians (M) and perpendicular to meridians (N) are used in calculations, both depend on the latitude.

$$M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\varphi\right)^{\frac{3}{2}}} \qquad N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}} \tag{2}$$

2.2. Map Projection

A map projection is defined by two functions.

$$x = x(\varphi,\lambda) \qquad y = y(\varphi,\lambda) \tag{3}$$

These functions define a transformation from the reference surface to the map plane, which is also called forward transformation. An inverse transformation is also needed, accordingly.

$$\varphi = \varphi(x,y) \qquad \lambda = \lambda(x,y) \tag{4}$$

The reference surface is undevelopable to the plane. Therefore the original surface is distorted after map projection transformation. The differential scale changes around a point that are called distortions are investigated by using a differential circle on the original surface and its projection that is an ellipse in general. According to Tissot semi-axes of this ellipse are perpendicular both on the original and projection surface. At the same time the semi-axes (m_1, m_2) correspond to the principal directions where the linear differential scale or linear distortion maximum and minimum (Richardus & Adler 1972). They are derived from partial derivatives of forward transformation equations (m_1: maximum, m_2: minimum). The formulae for unit sphere:

$$\begin{aligned}(m_1+m_2)^2 &= \left(\frac{\partial y}{\partial\varphi}+\frac{1}{\cos\varphi}\frac{\partial x}{\partial\lambda}\right)^2+\left(\frac{\partial x}{\partial\varphi}-\frac{1}{\cos\varphi}\frac{\partial y}{\partial\lambda}\right)^2\\(m_1-m_2)^2 &= \left(\frac{\partial y}{\partial\varphi}-\frac{1}{\cos\varphi}\frac{\partial x}{\partial\lambda}\right)^2+\left(\frac{\partial x}{\partial\varphi}+\frac{1}{\cos\varphi}\frac{\partial y}{\partial\lambda}\right)^2\end{aligned} \tag{5}$$

Linear distortions along meridians (h) and parallels (k) (unit sphere):

$$
\begin{aligned}
h &= \sqrt{\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2} \\
k &= \frac{1}{\cos\varphi}\sqrt{\left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2}
\end{aligned}
\tag{6}
$$

Maximum direction distortion (ω) and area distortion (p):

$$
\begin{aligned}
\sin\omega &= \frac{m_1 - m_2}{m_1 + m_2} \\
p &= m_1 m_2
\end{aligned}
\tag{7}
$$

3. Mercator and Web Mercator projections

The term Web Mercator is used to define a modified variant of Mercator projection used by web based mapping services such as Google Maps, Bing Maps, Open Street Map and etc.

3.1. Mercator projection

Mercator projection is one of the well-known projections, which is classed as cylindrical and conformal. The most known property of this projection is that the rhumb lines are depicted as straight lines. The length of the equator is preserved in the projection, which is also termed projection with one standard parallel. It is conformal, but distorts the areas towards poles too much. At 60° latitude areas are 4 times exaggerated. Since the poles go to the infinity in the projection plane, polar areas can not be shown. Despite not being suitable, it has been widely used for world maps.

The spherical Mercator projection is defined with the functions below.

$$
\begin{aligned}
x &= R\lambda \\
y &= R \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)
\end{aligned}
\tag{8}
$$

Since the projection is conformal, linear distortion (differential linear scale) is constant around a point.

$$
m = \frac{1}{\cos\varphi} \tag{9}
$$

If the projection is implemented with two standard parallels, area and distance distortions can be diminished in some extent. The projection equations and linear distortion are as follows (φ_0 being the latitude of the standard parallels in north and south).

$$x = R\cos\varphi_0\lambda$$
$$y = R\cos\varphi_0 \ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) \qquad (10)$$
$$m = \frac{\cos\varphi_0}{\cos\varphi}$$

The ellipsoidal Mercator projection equations with one standard parallel are below (Richardus & Adler 1972, Battersby 2014, NGA 2014):

$$x = a\lambda$$
$$y = a\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\left(\frac{1-e\sin\varphi}{1+e\sin\varphi}\right)^{\frac{e}{2}} \qquad (11)$$

Linear distortion is constant in any direction around a point.

$$m = \frac{a}{N\cos\varphi} \qquad (12)$$

The ellipsoidal projection can also be implemented with two standard parallels.

3.2. Web Mercator

The Web Mercator projection is a modified version of the Mercator projection, in which the WGS84 Ellipsoid (a=6378137.0 m, e^2= 0.00669438) is projected to the plane as if it were a sphere. The Cartesian coordinate system in the projection plane is arranged to meet the conventions in computer graphics, x right, y down (Fig.1). Because the poles can not be shown, the map at the smallest scale is arranged to be a square of 256x256 pixels. This map is assumed at the zoom level 0. The map size increases with the zoom level (n) (Google 2015a, Microsoft 2015).

$$x = \frac{128}{\pi}2^n(\lambda+\pi)$$
$$y = \frac{128}{\pi}2^n\left(\pi - \ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right) \qquad (13)$$

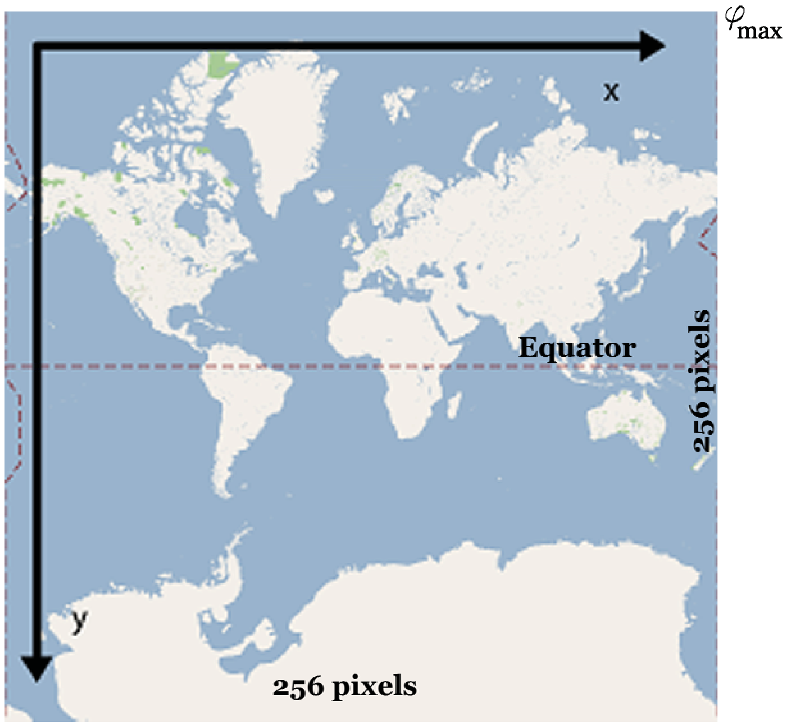

Figure 1. The Cartesian coordinate system in Web Mercator

Here the distance unit is pixel, of which physical size depends on the monitoring device. The maximum latitude can be obtained by setting y=0.

$$\ln\tan\left(\frac{\pi}{4}+\frac{\varphi_{\max}}{2}\right)=\pi$$
$$\varphi_{\max}=2\arctan e^{\pi}-\frac{\pi}{2} \qquad (14)$$

The numerical value is ±85.05129°, which is the north-most and south-most latitude of the map.

The spatial resolution of 1 pixel:

$$1\,pixel=\frac{\pi a}{2^{n+7}}\,meters \qquad (15)$$

The web mapping services use zoom levels up to 18, in general (e.g. Google Maps). The spatial resolution of 1 pixel ranges from 156 km at zoom level 0 to 0.60 m at zoom level 18.

Inverse projection:

$$\varphi = 2\arctan e^{\pi - \frac{\pi y}{2^{n+7}}} - \frac{\pi}{2}$$
$$\lambda = \frac{\pi x}{2^{n+7}} - \pi \tag{16}$$

Since the ellipsoid surface is projected onto the plane with spherical projection equations, Web Mercator projection is no more conformal. To evaluate the distortions occurred in this projection a metric Cartesian coordinate system is necessary. If we use a north-east xy system, where x corresponds to the zero-meridian and y to the equator, projection equations become as follows.

$$x = a\lambda$$
$$y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) \tag{17}$$

The principal directions at which linear distortion is maximum and minimum are coincident with the projected meridians and parallels.

$$h = \frac{1}{M}\frac{dy}{d\varphi} = \frac{a}{M\cos\varphi}$$
$$k = \frac{1}{N\cos\varphi}\frac{dx}{d\lambda} = \frac{a}{N\cos\varphi} \tag{18}$$

As seen from Eq. 18, the linear distortion is not constant which means that the projection is not conformal. Since $h > k$, linear distortion is maximum along meridians. Table 1 shows the projection distortions at certain latitudes. It is seen that the deviation from conformality decreases towards poles. The maximum direction distortion is maximal at the equator. The area distortion (p) is too high above 60°. In table 2, one degree meridian arc length in WGS84, Web Mercator and ellipsoidal Mercator are shown. The modification of the projection causes differences in Northing up to 9751 m, which is actually not a negligible value.

φ	h	k	p	ω
0°	1.006739	1.000000	1.006739	11' 32.7"
10°	1.021961	1.015324	1.037621	11' 11.9"
20°	1.070092	1.063761	1.138322	10' 11.9"
30°	1.159566	1.153734	1.337830	8' 40.0"
40°	1.308756	1.303601	1.706096	6' 47.1"
50°	1.556989	1.552665	2.417482	4' 46.8"
60°	1.998334	1.994973	3.986623	2' 53.6"
70°	2.917448	2.915150	8.504798	1' 21.3"
80°	5.741212	5.740046	32.95482	0' 21.0"
85°	11.43612	11.43554	130.7782	0' 5.3"

Table 1. Distortions in Web Mercator projection

φ	WGS84	Web M.	Mercator	Difference (WM-M)
0°	110574.389	111325.1	111312.0	13.1
10°	110611.187	113216.8	112942.0	274.8
20°	110710.615	118847.7	118310.8	536.8
30°	110860.926	129199.3	128396.9	802.4
40°	111044.261	146399.4	145318.3	1081.1
50°	111238.681	175017.9	173620.6	1397.2
60°	111420.728	226085.3	224268.1	1817.2
70°	111568.259	333556.5	330999.0	2557.5
80°	111663.201	675090.0	670299.7	4790.3
85°	111687.001	1424698.4	1414946.5	9751.9

Table 2. 1° meridian arc length on WGS 84, Web Mercator and Ellipsoidal Mercator and differences between Web Mercator and ellipsoidal Mercator (length unit: meters)

3.3. Latitude difference

One drawback of the Web Mercator implementation is the assumption of ellipsoidal latitude as if it were spherical. There is a difference between ellipsoidal and spherical latitudes. Web Mercator projection uses a sphere of which radius equals the equatorial radius of WGS84 (a). The spherical latitude, where R=a, is approximately equals to the central latitude of the ellipsoid (Figure 2).

$$\tan \gamma \cong \left(1 - e^2\right) \tan \varphi \tag{19}$$

If spherical calculations are performed with latitudes taken from a web mapping service such as Google Maps, the differences should be taken into account.

The geometry library of the Google Maps API ignores this difference in latitude, and performs the spherical calculations with ellipsoidal latitude as if it were spherical (Google 2015c).

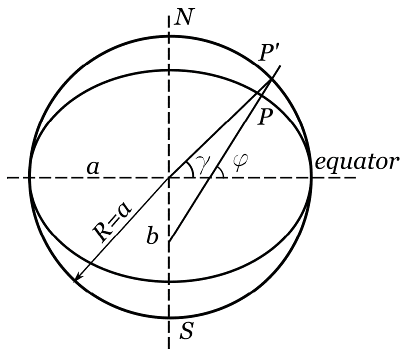

Figure 2. Ellipsoidal and spherical latitude

If the central latitude was used in the Web Mercator projection, the resulting projection would be conformal.

4. An Alternative solution: virtual globe representation

It is obvious that either Web Mercator or ellipsoidal Mercator is not a good choice for web mapping, because of high scale changes towards poles. The geodesic lines are too distorted in long distances. People who are aware of projection distortions can easily understand why geodesic lines are longer than the straight line connecting points of interest. Figure 3 shows a geodesic line connecting Istanbul and New York in Goggle Maps and Google Earth. The appearance is too different in map and virtual globe.

Virtual globe view is a good alternative to Web Mercator. In Google Maps API v2, now deprecated, switching to Google Earth View was possible. This option is no more supported in the current version (v.3). Google launched Google Earth API that make possible to have Google Earth within a web site, as a mashup. This API was functional under certain platforms with a plug-in. It is deprecated lately (Google 2015b). Bildirici et al. (2014) combined Google Maps API and Google Earth API for educational purposes within a single map mashup. Their mashup made possible to compare rhumb-lines and great circles on the map and on the globe.

By using virtual globe representation the Polar Regions are shown, which is not possible in Web Mercator. Web mapping services should make possible globe views with map and satellite options to avoid misunderstandings caused by Web Mercator. It would be a good opportunity for inexperienced users in terms of map projections.

Figure 3. A geodesic line connecting Istanbul and New York in Google Maps and Google Earth (8072 km)

5. Conclusion

Web Mercator projection as a variant of well-known Mercator projection becomes a de facto standard widely used by online mapping services such as Google Maps, Bing Maps etc. Despite being unsuitable in terms of map projection distortions, it is beneficial in terms of computer graphics and tile system that makes possible to realize different zoom levels of map data. It is especially important that the graticule appears rectangular that matches the Cartesian coordinates on the map plane. Only cylindrical projections enable such an opportunity. Virtual globe representations are another way of web-based visualization. Google supported earth view in the previous version of Google Maps API, and later launched Google Earth API. Both are deprecated now. Since globe representation is a 3D visualization not affected by map projection distortions, their use is beneficial together with 2D map. Such possibilities can help map users to understand changes in geometry caused by map projection.

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