Some remarks on the question of pseudocylindrical projections with minimum distortions for world maps

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Extended Abstract

Atlases for the general public and for schools cannot miss chorographic world maps. The traditional expectations made on the planar representations of the known world – the symmetry, the scale invariance and the similarity – also apply to them. The latter two requirements should imply the smallest possible map distortions (the changes of lengths, angles and areas, which emerge during mapping) and on the other hand, the similarity of the outline of the mapped Earth.

In the aphylactic (neither conformal nor equal-area) projections both the angular and area distortions can be reduced by the principle of „balance of errors", using the Airy-Kavrayskiy criterion. This favourable index number gives the aggregated measure of the sum of angular and area distortions for the territory of the World. The pseudocylindrical map projections are often used in the field of representation of the Earth. The purpose of this paper is to construct some of such map projections which minimze this criterion, while the outline of the mapped Earth keeps an oval shape.

This task demands the determination of the appropriate „projection equations" $x(\varphi,\lambda)$ and $y(\varphi)$, which carry out the mapping of the curved Earth surface (parametrized by the geographic latitude φ and longitude λ) to the map plane (parametrized by the rectangular coordinates x and y). The minimization of the Airy-Kavrayskiy criterion is a mathematical problem known from the calculus of variations. The task was solved in two steps. The first one was directed to the function $y(\varphi)$, and the result was that the fairly good projection has to have true scale midmeridian without respect to the function $x(\varphi,\lambda)$ and the territory to be represented. The second step results in the right function $x(\varphi,\lambda)$ by the direct minimization of the above

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mentioned criterion, and the calculations used some different kinds of non-linear functions. The numerical values for the coefficients c_i of $x(\varphi,\lambda)$ were obtained with the help of the downhill simplex method.

The outline of the mapped Earth is favourable from the aspect of similarity, if the poles are represented as single points instead of lines, moreover, the curvature of the outline does not vary extremely in the environment of the poles. If we take into consideration both the value of the Airy-Kavrayskiy criterion and the change of the value of the curvature κ along the outline, we get some favourable projection solutions depending on the kind of the approximating function. These projections were introduced in this paper.

One of the preferred versions is the following:

The projection equations:

$$x = c_1 \cdot \left(1 + c_2 \cdot \varphi^2\right) \cdot \sqrt{1 - \left(\frac{2 \cdot \varphi}{\pi}\right)^2} \cdot \lambda$$

$$y = \varphi$$

where c_1=0.768, c_2=0.170; the parallels are equidistant.

The value of the Airy-Kavrayskiy criterion: 0.13522;

$\kappa_{\pm 90^\circ}$=0.134 $\kappa_{\pm 85^\circ}$=0.221

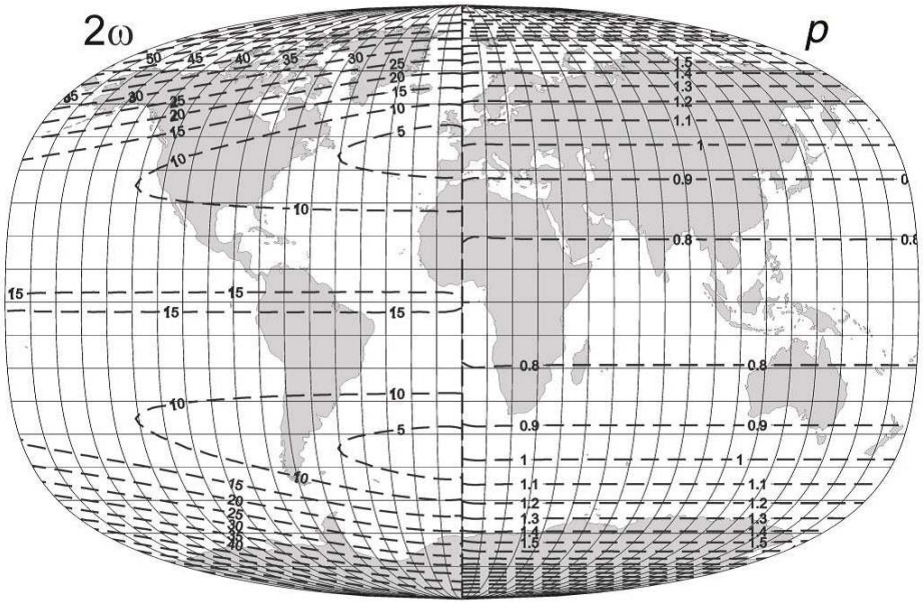

Figure 1 shows the isolines of the maximum angular deformation 2ω and the area scale p.